Ore localization, associated torsion and algorithms

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Localizing a commutative ring at a multiplicatively closed subset is an important and well-understood tool in the study of commutative rings. For non-commutative domains, the concept of Ore localization at (left) Ore sets introduced by Øystein Ore ([1]) is a generalization that retains most of the properties of classical localization. Its most prominent application in the context of algebras of operators is the formalism of passing from polynomial to rational coefficients.

Building on previous results in the commutative setting well-known to Zariski and Samuel ([3]), we present a new canonical form for Ore sets that gives full insight into the structure of the associated localization and its units.

As a further application of Ore sets we introduce the concept of local torsion of modules over (non-commutative) domains, which results in a finer description of the torsion structure of a module.

In the case of finitely presented modules, local torsion is closely related to S-closure, i. e. the closure of submodules of free modules with respect to an Ore set, which is an instance of a more general construction that also encompasses the canonical form for Ore sets above.

Furthermore, we show a connection between S-closure and contraction of ideals in the localization to the unlocalized ring. There are several algorithmic approaches to the latter, most notably Weyl closure ([2]). We expand the algorithmic toolbox by a variation of a commutative algorithm that computes S-closure in a restricted setting which is of importance in the theory of D-modules.

References

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