Local Closure of Ore Algebras

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Let K be a field, $K[x] = K[x_1, \ldots, x_n]$ the polynomial ring in n variables and $R := K[x][\partial; \mathrm{id}, \delta]$ an Ore extension of $K[x]$ with a derivation δ. This is an algebraic model for linear ODEs with the x_i taking the role of coefficient functions and δ modelling the derivative of those functions. Here the elements and ideals in R represent ODEs and systems of ODEs respectively. In this setting the contraction ideal

$$\mathrm{Cont}(I) := (K[x] \setminus \{0\})^{-1} I \cap R$$

for an ideal $I \subseteq R$ is the largest ideal in R that has the same solutions as I (on a suitably chosen open subset of the complex plane). The problem of computing the contraction ideal for a given ideal I has been solved e.g. in the Weyl algebra (see [1]) under the name of Weyl closure, but the general case remains unsolved.

If we restrict ourselves to principal ideals $\langle f \rangle \subseteq R$ it is enough to consider the local closure

$$\mathrm{Cl}_p(f) := \{p^{-k}\} \langle f \rangle \cap R$$

at the leading ∂-coefficient p of f. This relates to the problem of desingularization of differential operators, which has for example been considered (for $K[x] = K[x_1]$) by [2] or for a fixed maximal degree of the desingularizing operator by [3].

We will present an approach that can be used in the general case to compute all elements $h \in R$ such that $\frac{1}{p} hf \in R$ for a given $p \in K[x]$, i.e. all desingularizing operators that have degree 1 in $\frac{1}{p}$. Such h have some significance as $\mathrm{Cl}_p(f)$ is strictly larger than $\langle f \rangle$ if and only if the set of all $h \in R$ with $\frac{1}{p} hf \in R$ is strictly larger than pR. This means that we can test whether p is removable from f. Furthermore if p is removable from f there is always a p-removing operator of minimal degree in ∂ that has the form $\frac{1}{p} h$.

References

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[3] S. Chen, M. Kauers, M. Singer, *Desingularization of Ore operators*, Journal of Symbolic Computation, pp. 617-626 (Mai-June 2016).