

Tensor reduction systems for operator algebras and normal forms

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Ore algebras are a well-established algebraic tool for an algorithmic treatment of many common cases, like differential and difference operators. However, integral operators cannot be constructed that way, for example. We use our two-level generalization [2] of Bergman’s basis-free setting in tensor algebras [1] that allows for smaller reduction systems and tends to make computations more efficient. In this setting, we present a heuristic analog of Buchberger’s algorithm for computer-assisted construction of Gröbner bases starting from basic identities of operators. We illustrate it and the package TenReS [3] using integro-differential operators with linear substitutions as example. These operator algebras have applications to delay equations [4] and address the univariate case of [5].

References

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