Thomas Decomposition and Nonlinear Control Systems

D. Robertz[1]

[1] *Centre for Mathematical Sciences, Plymouth University, 2-5 Kirkby Place, Drake Circus, Plymouth PL4 8AA, UK, daniel.robertz@plymouth.ac.uk*

This talk presents joint work with Markus Lange-Hegermann. We apply the Thomas decomposition technique to nonlinear control systems, in particular to the study of the dependence of the system behavior on parameters. Thomas' algorithm is a symbolic method which splits a given system of nonlinear partial differential equations into a finite family of so-called simple systems which are formally integrable and define a partition of the solution set of the original differential system. Different simple systems of a Thomas decomposition describe different structural behavior of the control system in general. We give a short introduction to the Thomas decomposition method and show how notions such as invertibility, observability and flat outputs can be studied. A Maple implementation of Thomas' algorithm is used to illustrate the techniques on explicit examples.

References

[1] T. Bächler, V. P. Gerdt, M. Lange-Hegermann and D. Robertz, *Algorithmic Thomas Decomposition of Algebraic and Differential Systems*, J. Symbolic Comput. **47**, 10, pp. 1233–1266 (2012).

[2] T. Bächler and M. Lange-Hegermann, `AlgebraicThomas` *and* `DifferentialThomas`*: Thomas decomposition of algebraic and differential systems*, freely available at `http://wwwb.math.rwth-aachen.de/thomasdecomposition`.

[3] G. Conte, C. H. Moog and A. M. Perdon, *Nonlinear control systems*, vol. 242 of Lecture Notes in Control and Information Sciences, Springer, London, 1999.

[4] S. Diop, *Elimination in control theory*, Math. Control Signals Systems **4**, 1, pp. 17–32 (1991).

[5] S. Diop, *Differential-algebraic decision methods and some applications to system theory*, Theoret. Comput. Sci. **98**, 1, pp. 137–161 (1992).

[6] M. Fliess and S. T. Glad, *An Algebraic Approach to Linear and Nonlinear Control*, in: H. L. Trentelman and J. C. Willems (eds.), *Essays on Control: Perspectives in the Theory and its Applications*, pp. 223–267, Birkhäuser, Boston, 1993.

[7] M. Fliess, J. Lévine, P. Martin and P. Rouchon, *Flatness and defect of non-linear systems: introductory theory and examples*, Internat. J. Control **61**, 6, pp. 1327–1361 (1995).

[8] V. P. Gerdt, *On decomposition of algebraic PDE systems into simple subsystems*, Acta Appl. Math. **101**, 1-3, pp. 39–51 (2008).

[9] M. Lange-Hegermann and D. Robertz, *Thomas decompositions of parametric nonlinear control systems*, in: Proceedings of the 5th Symposium on System Structure and Control, Grenoble, France, pp. 291–296, 2013.

[10] M. Lange-Hegermann and D. Robertz, *Thomas Decomposition and Nonlinear Control Systems*, submitted for publication.

[11] H. Nijmeijer and A. van der Schaft, *Nonlinear dynamical control systems*, Springer, New York, 1990.

[12] V. Levandovskyy and E. Zerz, *Obstructions to genericity in study of parametric problems in control theory*, in: H. Park and G. Regensburger (eds.), *Gröbner bases in control theory and signal processing*, vol. 3 of Radon Ser. Comput. Appl. Math., pp. 127–149. Walter de Gruyter, Berlin, 2007.

[13] J.-F. Pommaret, *Partial differential control theory*, vol. 530 of Mathematics and its Applications, Kluwer, Dordrecht, 2001.

[14] J.-F. Pommaret and A. Quadrat, *Formal obstructions to the controllability of partial differential control systems*, in: Proceedings of IMACS, Berlin, Germany, vol. 5, pp. 209–214, 1997.

[15] D. Robertz, *Formal Algorithmic Elimination for PDEs*, vol. 2121 of Lecture Notes in Mathematics, Springer, Cham, 2014.

[16] J. M. Thomas, *Differential Systems*, vol. XXI of American Mathematical Society Colloquium Publications, American Mathematical Society, New York, N. Y., 1937.