Algebraic Theory of Linear Partial Differential Algebraic Equations

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We discuss general, i. e. also under- and overdetermined, systems of linear partial differential equations (sometimes also called linear partial differential algebraic equations) using algebraic techniques like Gröbner or involutive bases. The main emphasis is on the construction of formally well-posed initial value problems where for every choice of formal power series as initial data a unique formal power series solution exists. This problem is essentially equivalent to finding complementary combinatorial decompositions for monomial modules. In addition, we show how the theory of Gröbner bases for ideals of linear differential operators leads natural to index concepts that can be directly defined for systems of partial differential equations without the need to reduce first to ordinary differential algebraic equations (e. g. via semi-discretisations or integral transforms).

References

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