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Effective criterion to test differential transcendence of special functions.

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Consider a field $\mathbf{k}$ equipped with an automorphism ϕ. Typical examples are

- $\mathbf{k} = \mathbb{C}^{\mathbb{Z}}$, $\phi(u_n) := (u_{n+1})$;
- $\mathbf{k} = \mathbb{C}(x)$, $\phi f(x) := f(x+1)$;
- $\mathbf{k} = \mathbb{C}(x)$, $\phi f(x) := f(qx)$, $q \in \mathbb{C}^*$;
- $\mathbf{k} = \cup_{\ell \in \mathbb{N}^*} \mathbb{C}(x^{1/\ell})$, $\phi f(x) := f(x^p)$, $p \in \mathbb{N}^*$.

A difference equation is a linear equation of the form

$$a_0 y + \cdots + a_n \phi^n(y) = 0,$$

with $a_0, \ldots, a_n \in \mathbf{k}$. The difference Galois theory, see [1], attaches to such equation a linear algebraic subgroup of $\mathrm{GL}_n(\mathbb{C})$ that measures the algebraic relations among the solutions of the difference equation. More recently, it has been developed in [2] a Galois theory that aims at understanding the algebraic and differential relations among the solutions of the difference equation

The goal of this talk is to give explicit and computable criterias to ensure that a solutions of an order two difference equation does not satisfy any algebraic differential equations in coefficients in $\mathbf{k}$. We apply this criterion to the elliptic analogue of the hypergeometric functions.

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