

## Symbolic computation for integro-differential-time-delay operators with matrix coefficients

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In order to facilitate symbolic computations with systems of linear functional equations, we require an algebraic framework for such systems which enables effective computations in corresponding rings of operators. We briefly explain the recent developed tensor approach from scalar equations [1] to the matrix case [2], by allowing noncommutative coefficients. Noncommutative coefficients even allow to handle systems of generic size. Normal forms are a key ingredient for computing with operators and rely on a confluent reduction system.

The tensor approach is flexible enough to cover many operators, like integral operators, that do not fit the well established framework of skew-polynomials. For instance, it can be used to construct the ring of integro-differential operators with linear substitutions (IDOLS) having (noncommutative) matrix coefficients, containing the ring of integro-differential-time-delay operators. In the *Mathematica* package `TenRes` we provide support for tensor reduction systems [3]. In addition, we implement the ring of IDOLS and corresponding normal forms. We illustrate how, by elementary computations in this framework, results like the method of steps can be found and proven in an automated way. We also apply normal forms of IDOLS to partly automatize certain computations related to differential time-delay systems, e.g. Artstein's transformation [4] and its generalization [5].

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## References

- [1] J. HOSSEIN POOR; C. G. RAAB; G. REGENSBURGER, *Algorithmic operator algebras via normal forms for tensors*. In *Proceedings of ISSAC 16*, pp. 397–404, 2016.
- [2] J. HOSSEIN POOR; C. G. RAAB; G. REGENSBURGER, *Algorithmic operator algebras via normal forms in tensor rings, 33 pages*. *Journal of Symbolic Computation*, **85**, pp. 247–274, 2018.
- [3] J. HOSSEIN POOR; C. G. RAAB; G. REGENSBURGER *Normal forms for operators via Gröbner bases in tensor algebras*, In *Proceedings of ICMS 2016*, volume 9725 of *LNC3*. Springer, pp. 505–513, 2016.
- [4] A. QUADRAT, *A constructive algebraic analysis approach to Artstein's reduction of linear time-delay systems*, In *IFAC-PapersOnLine*, 48 (12), pp. 209–214, 2015.
- [5] T. CLUZEAU; J. HOSSEIN POOR; A. QUADRAT; C.G. RAAB; G. REGENSBURGER, *Symbolic computation for integro-differential-time-delay operators with matrix coefficients*, 6 pages, 2018, submitted.

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