Applications of Computer Algebra – ACA2018
Santiago de Compostela, June 18–22, 2018

Computer algebra and the Lanczos problems in arbitrary dimension

J.-F. Pommaret[1]

When $\mathcal{D}$ is a linear partial differential operator of any order, a direct problem is to look for an operator $\mathcal{D}_1$ generating the compatibility conditions (CC) $\mathcal{D}_1\eta = 0$ of $\mathcal{D}\xi = \eta$. We may thus construct a differential sequence with successive operators $\mathcal{D}, \mathcal{D}_1, \mathcal{D}_2, ...$, where each operator is generating the CC of the previous one. Introducing the formal adjoint $ad()$, we have $\mathcal{D}_i \circ \mathcal{D}_{i-1} = 0 \Rightarrow ad(\mathcal{D}_{i-1}) \circ ad(\mathcal{D}_i) = 0$ but $ad(\mathcal{D}_{i-1})$ may not generate all the CC of $ad(\mathcal{D}_i)$. When $D = K[d_1, ..., d_n] = K[d]$ is the (non-commutative) ring of differential operators with coefficients in a differential field K, it gives rise by residue to a differential module M over D. The homological extension modules $ext^i(M) = ext^i_D(M, D)$ with $ext^0(M) = hom_D(M, D)$ only depend on M and are measuring the above gaps, independently of the previous differential sequence.

The purpose of this talk is to explain how to compute extension modules for certain Lie operators involved in the formal theory of Lie pseudogroups in arbitrary dimension n. In particular, we prove that the extension modules highly depend on the Vessiot structure constants c. When one is dealing with a Lie group of transformations or, equivalently, when $\mathcal{D}$ is a Lie operator of finite type, then we shall prove that $ext^i(M) = 0, \forall 0 \leq i \leq n-1$. It will follow that the Riemann-Lanczos and Weyl-Lanczos problems just amount to prove such a result for $i = 2$ and arbitrary n when $\mathcal{D}$ is the Killing or conformal Killing operator. We finally prove that $ext^i(M) = 0, \forall i \geq 1$ for the Lie operator of infinitesimal contact transformations with arbitrary $n = 2p + 1$. Most of these new results have been checked by means of computer algebra.

Keywords: Differential sequence, Variational calculus, Differential constraint, Control theory, Killing operator, Riemann tensor, Bianchi identity, Weyl tensor, Lanczos tensor, Contact transformations, Vessiot structure equations

[1]CERMICS,
Ecole des Ponts ParisTech,
France
`jean-francois.pommaret@wanadoo.fr`