

## Algebraic proofs of operator identities

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Many interesting properties of linear operators can be phrased as operator identities, which then can be proven algebraically. In practice, however, linear operators often map between different spaces, then we can no longer add or compose any two such operators. For instance, this already happens with rectangular matrices or with differential operators having rectangular matrix coefficients.

In order to still be able to do meaningful symbolic computations with such operators on the computer, an algebraic framework is needed that deals with the corresponding domains and codomains of operators when adding and multiplying operators. In principle, symbolic computation with such operators (or matrices) would require at each step taking care of the domains and codomains of those operators (or of the formats of the matrices). In contrast, we aim at an a-posteriori justification of an identity, independent of how it was computed algebraically.

In this talk we present first results towards such an algebraic framework based on quivers and noncommutative Gröbner bases, which could be applied to operators with rectangular matrix coefficients. We will also present examples from the theory of generalized inverses using noncommutative Gröbner bases.

**Keywords:** Linear operators, noncommutative Gröbner bases

## References

- [1] J. HOSSEIN POOR; C. G. RAAB; G. REGENSBURGER, Algebraic proofs of operator identities. In preparation, 2018.

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