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Desingularization in the q-Weyl algebra

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The desingularization problem has been primarily studied for linear differential operators with polynomial coefficients. The solutions of such an equation are called *D-finite* functions. It is well known that a singularity at a certain point x_0 of one of the solutions must be reflected by the vanishing (at x_0) of the leading coefficient of the differential equations. However, the converse however is not always true: not every zero of the leading coefficient polynomial induces a singularity of at least one function in the solution space. The purpose of desingularization is to construct another equation, whose solution space contains that of the original equation, and whose leading coefficient vanishes only at the singularities of the previous solutions. Typically, such a desingularized equation will have a higher order, but a lower degree for its leading coefficient. In summary, desingularization provides some information about the solutions of a given differential equation.

The authors of [1, 3] give general algorithms for the Ore case. However, from a theoretical point of view, the story is not yet finished, in the sense that there is no order bound for desingularized operators in the Ore case. We consider the desingularization problem in the first q-Weyl algebra. Our main contribution is to give an order bound for desingularized operators, and thus derive an algorithm for computing desingularized operators in the first q-Weyl algebra. In addition, an algorithm is presented for computing a generating set of the first q-Weyl closure of a given q-difference operator. As an application, we certify that several instances of the colored Jones polynomial from knot theory are Laurent polynomial sequences by computing the corresponding desingularized operator.

Keywords: Desingularization, q-Weyl algebra, Knot Theory

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