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An effective version of Warfield's theorem

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A linear multidimensional system of q equations with p unknown functions $\eta_1, \cdots, \eta_p$ maybe described by a matrix $R \in D^{q\times p}$ as follows:

$$\ker_{\mathcal{F}}(R.) := \{\eta \in \mathcal{F}^p \mid R\eta = 0\}, \tag{1}$$

where $\mathcal{F}$ is the functionnal space where we are looking for the solutions. The latter admits a structure of *left D-module*, which enables us to described the space of solutions in terms of module theory: $\ker_{\mathcal{F}}(R.) \simeq \hom_D(M, \mathcal{F})$, where $M = D^{1\times p}/(D^{1\times Q}R)$ is the left D-module *finitely presented* by the matrix R. Under this point of view, some structural properties of (1) can be studied by mean of algebraic invariants. In particular, the formal manipulation of the system, such as exchange lines, multiply lines by a constant, lead to study the links between matrix conjugation and module isomorphisms. A result due to Fitting [2], asserts that two matrices presenting isomorphic left D-modules can be enlarged by blocks of 0 and identities to get equivalent matrices. A result due to Warfield [3] asserts that the number of 0 and identity blocs in the result of Fitting maybe reduced, the resulting matrices are still equivalent. This reduction procedure is based on the properties of the *stable rank* of D. The purpose of this talk is to provide an effective version of the Warfield's result. For that, we begin with the effective version of Fitting's result given in [1], and we use the stable rank for reducing the number of 0 and identity blocs.

Keywords
Module isomorphisms, equivalent matrices, stable rank

References
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