

Some properties and applications of multivariate dimension polynomials and their computation in Python *

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In this presentation we consider Hilbert-type polynomials in several variables that characterize finitely generated differential modules, that is, modules over rings of differential operators over differential fields. Such a polynomial describes the dimensions of components of a natural p -dimensional filtration ($p \geq 2$) associated with a system of generators of the module and a partition of the basic set of derivations into p subsets. Multivariate dimension polynomials of differential modules were introduced in [3] where their existence was established with the use of the technique of characteristic sets. The results of this work were essentially improved in [4] where one can find methods of computation of multivariate dimension polynomials via constructing generalized Gröbner bases (i. e., Gröbner bases with respect to several term orderings) in free differential modules. This approach was extended in [5] and [1] where the authors introduced a concept of relative Gröbner bases (Gröbner bases with respect to two generalized term orderings) and applied it to the computation of bivariate difference-differential dimension polynomials. There are also several recent works with similar results on multivariate dimension polynomials of difference and inversive difference modules.

The main results of our talk are as follows. We present algorithms for computing generalized Gröbner bases in free differential modules and for computing multivariate differential dimension polynomials, as well as implementations of these algorithms in Python. We also present some conditions under which a multivariate differential dimension polynomial has a special simple form. The obtained results are applied to the computation of differential dimension polynomials associated with the advection-diffusion equation and PDEs that arise in mathematical models of ion exchange chromatography studied in [2].

Keywords

Differential field, Differential module, Generalized Gröbner basis, Differential dimension polynomial

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