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The Kernel-Method and Automated Positive Part Extraction

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A lattice walk is a sequence $P_0, P_1, \dots, P_n$ of points in $\mathbb{N}^d$. The points P_0 and P_n are its starting and end point, respectively, n is its length, and the consecutive differences $P_{i+1} - P_i$ are its steps. Fixing a starting point P and a set S of admissible steps combinatorialists ask for the number $f(Q, n)$ of walks in $\mathbb{N}^d$ that start at P, consist of n steps, all taken from S, and end at Q: Are there nice formulas for these numbers? What is their asymptotics as n goes to infinity? In answering these questions it is helpful to study the associated generating function

$$F(x,t) = \sum_{n \geq 0} \left(\sum_{P \in \mathbb{N}^d} f(P,t) x^P \right) t^n \in \mathbb{Q}[x][[t]]$$

and the functional equation it satisfies and to decide whether it satisfies a linear differential equation with polynomial coefficients or not. Mishna [1], [2] and Bousquet-Mélou [2] initiated a systematic study of this problem for walks restricted to $\mathbb{N}^2$ whose steps are taken from a subset S of $\{-1, 0, 1\}^2$ and introduced a method involving elementary power series algebra for proving D-finiteness of the generating functions of some instances of this problem. Bousquet-Mélou et al. [3] generalized it to walks with steps not necessarily restricted to $\{-1, 0, 1\}^2$. We show how this method can be extended and automatized using Gröbner bases and a generalized Newton-Puiseux algorithm.

References

[1] MARNI MISHNA, *Classifying Lattice Walks Restricted to the Quarter Plane.* Journal of Combinatorial Theory, Series A Vol 116(2): 460–477, 2009.
[2] MARNI MISHNA, MIREILLE BOUSQUET-MÉLOU. *Walks with Small Steps in the Quarter Plane.* Algorithmic Probability and Combinatorics, Special Volume of the Contemporary Mathematics Series of the AMS 520, 1–40, 2010.
[3] ALIN BOSTAN, MIREILLE BOUSQUET-MÉLOU, STEVEN MELCZER, *Counting Walks with Large Steps in an Orthant.* Journal of the European Mathematical Society, to appear.