

Linear PDE with constant coefficients

Marc Härkönen¹, Rida Ait El Manssour², Bernd Sturmfels^{2,3} [harkonen@gatech.edu]

¹ School of Mathematics, Georgia Institute of Technology, Atlanta, USA

² Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany

³ Department of Mathematics, University of California at Berkeley, Berkeley, USA

In an undergraduate differential equations course we learn to solve a homogeneous linear ordinary differential equation with constant coefficients by finding roots of its characteristic polynomial. Thus the problem of solving an ODE is reduced to factoring a univariate polynomial. This simple idea was generalized in the 1960s for systems of linear PDE. The celebrated Fundamental Theorem by Ehrenpreis and Palamodov asserts that all solutions to a system of PDE can be represented by a finite sum of integrals over certain algebraic variety. This representation is strongly connected to the geometry of schemes or coherent sheafs corresponding to polynomial ideals or modules. In this talk I will review some of the main historical results, along with some recent advances in symbolic and numerical algorithms.

Keywords

Partial Differential Equations, Algebraic Geometry, Numerical Algebraic Geometry

References

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