

Towards a Theory of Domains for Harmonic Functions and its Symbolic Counterpart

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In this talk, a sequel of [6], a theory of Domains for Harmonic Sums is proposed.

We begin by reviewing the calculus induced by the framework of [6]. In there, we extended Polylogarithm functions over a subalgebra of noncommutative rational power series, recognizable by finite state (multiplicity) automata over the alphabet $X = \{x_0, x_1\}$ (see [6]). The stability of this calculus under shuffle products relies on the nuclearity of the target space (see [17]). We also concentrated on algebraic and analytic aspects of this extension allowing to index polylogarithms, at non positive multi-indices, by rational series and also allowing to regularize divergent polyzetas, at non positive multi-indices (see [6]).

In this talk, as a continuation of works in [6] and in order to understand the bridge between the extension of this “polylogarithmic calculus” and the world of harmonic sums, we propose a local theory, adapted to a full calculus on indices of Harmonic Sums based on the Taylor expansions, around zero, of polylogarithms with index x_1 on the rightmost end. This theory is not only compatible with Stuffle products but also with the Analytic Model. In this respect, it provides a stable and fully algorithmic model for Harmonic calculus. Examples by computer are also provided.

Keywords

Generating series, Taylor expansion, Asymptotic expansion, Regularization.

References

- [1] J. BERSTEL, C. REUTENAUER, *Rational series and their languages*, Spr.-Ver., 1988.
- [2] J. BERSTEL, C. REUTENAUER, *Noncommutative Rational Series with Applications*, Encyclopedia of Mathematics and its Applications series, Cambridge University Press:248 pages, 2011.

- [3] C. COSTERMANS; V. HOANG NGOC MINH, *Some Results à l'Abel Obtained by Use of Techniques à la Hopf*, Global Integrability of Field Theories and Applications, Daresbury, 2006.
- [4] C. COSTERMANS; V. HOANG NGOC MINH, *Noncommutative algebra, multiple harmonic sums and applications in discrete probability*, Journal of Symbolic Computation, 801–817, 2009.
- [5] BUI VAN CHIEN; V. HOANG NGOC MINH; NGO QUOC HOAN, *Families of eulerian functions involved in regularization of divergent polyzetas*, in preparation, 2020.
- [6] G.H.E. DUCHAMP; V. HOANG NGOC MINH; NGO QUOC HOAN, *Kleene stars of the plane, polylogarithms and symmetries*, Theoretical Computer Science, (800):52–72, 2019.
- [7] G.H.E. DUCHAMP; V. HOANG NGOC MINH; V. NGUYEN, *Towards a noncommutative Picard-Vessiot theory*, in preparation, 2020.
- [8] B. V. DRINFELD, *On quasitriangular quasi-Hopf algebra and a group closely connected with $Gal(\mathbb{Q}/\mathbb{Q})$* , Leningrad Math. J. , (4):829–860, 1991.
- [9] J. HADAMARD, *Théorème sur les séries entières*, Acta Mathematica, 22:55– 63, 1899.
- [10] V. HOANG NGOC MINH, *Summations of polylogarithms via evaluation transform*, Math. & Comput. Simul., 1336:707–728, 1996.
- [11] V. HOANG NGOC MINH, *Differential Galois groups and noncommutative generating series of polylogarithms*, in Automata, Combinatorics and Geometry, 7th World Multi-conference on Systemics, Cybernetics and Informatics, Florida, 2003.
- [12] V. HOANG NGOC MINH, *Finite polyzêtas, Poly-Bernoulli numbers, identities of polyzêtas and noncommutative rational power series*, Proc. of 4th International Conference on Words, pages 232–250, 2003.
- [13] V. HOANG NGOC MINH, *On the solutions of universal differential equation with three singularities*, Confluentes Mathematic, 11, no. 2:25–64, 2019.
- [14] V. HOANG NGOC MINH; G. JACOB; N.E. OUSSOUS; M. PETITOT, *Aspects combinatoires des polylogarithmes et des sommes d'Euler-Zagier*, Journal électronique du Séminaire Lotharingien de Combinatoire, B43e, 2000.
- [15] P. MONTEL, *Leçons sur les familles normales de fonctions analytiques et leurs applications*, Gauthier-Villars, 2010.
- [16] RICHARD P. STANLEY, *Enumerative Combinatorics*, Cambridge University Press, Vol. I, 1997.
- [17] H. H. SCHAEFER AND M. P. WOLFF, *Topological Vector Spaces*, Springer-Verlag New York, 1999.