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Automatizing proofs of properties of operators

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Recently, two of the authors have developed a framework to rigorously prove statements about matrices and linear operators in a purely algebraic fashion [1]. To this end, operators are modelled as noncommutative polynomials and restrictions imposed by the domains and codomains of these operators are encoded in a labeled quiver. Then, proving that an identity of operators follows from other identities translates into verifying ideal membership of noncommutative polynomials and showing *compatibility* of certain polynomials with a quiver.

In this talk, we illustrate how this framework can be applied to problems in several different branches of mathematics. In particular, we discuss how to handle properties of operators that cannot be expressed in form of a single identity, such as quasi-identities or existential statements. For example, we show fully automated proofs of statements about Moore Penrose inverses as also published in [2] as well as of well-known theorems from homological algebra such as the five lemma. Furthermore, we present the MATHEMATICA package `OperatorGB` [3], which provides extensive support for proving properties of matrices and operators along the lines of the framework, and give a brief overview of the underlying algorithms implemented.

Keywords
matrices and linear operators, algebraic operator identities, automated proofs, noncommutative polynomials, quiver representations

References
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