

Multivariate Bernstein-type Polynomials of Finitely Generated D -Modules

Alexander Levin

[levin@cua.edu]

Department of Mathematics, The Catholic University of America, Washington, DC 20064, USA

In [1], I. Bernstein introduced an analog of the Hilbert polynomial for a finitely generated filtered module over a Weyl algebra $A_n(K)$, the ring of differential operators over the polynomial ring $K[x_1, \dots, x_n]$ generated by the partial derivations $\partial/\partial x_i$ ($1 \leq i \leq n$). He also developed the theory of multiplicity for the class of such modules (called D -modules) and obtained interesting analytical applications of this theory (many of them are considered in Björk’s book [2]).

In this presentation we prove the existence, determine invariants, and outline methods of computation of multivariate Bernstein-type polynomials of finitely generated D -modules associated with partitions of the basic sets of indeterminates and derivations. We show that such polynomials not only characterize the Bernstein class of left $A_n(K)$ -modules, but also carry, in general, more invariants than the univariate Bernstein dimension polynomial. The presented results generalize the main results of [3] and give new properties of filtered D -modules and associated dimension polynomials.

Keywords

Weyl algebra, D -module, Dimension polynomial

References

- [1] I. N. BERNSTEIN, Modules over the ring of differential operators. A study of the fundamental solutions of equations with constant coefficients. *Funct. Anal. and its Appl.* **5**, 89–101 (1971).
- [2] J.-E. BJÖRK, *Rings of Differential Operators*. North Holland Publishing Co., Amsterdam, 1979.
- [3] C. DÖNCH; A. LEVIN, Bivariate Dimension Polynomials and New Invariants of Finitely Generated D -modules. *Int. J. Algebra Comput.* **23** (7), 1625–1651 (2013).