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Commutative rings of Differential Operators

Emma Previato[1] [ep@math.bu.edu]

[1] Department of Mathematics and Statistics, Boston University, Boston, MA, USA

Maximal-commutative algebras of ordinary differential operators (ODOs), equivalently, centralizers of a given ODO, are a topic of current interest and many open problems. The notion of rank plays a major role. In the Weyl algebra W, namely ODOs with polynomial coefficients, the problems become more difficult. This talk will review the Dixmier test [1] and present its computational implementation [3] in W, for the case study of centralizers of 4th order ODOs. The algorithm, automated in Maple 18, starts with an ODO L of order 4 (in normalized form) and, provided the centralizer is non-trivial, by iterating the division algorithm finds a commuting operator B such that the pair $L,\ B$ is a basis of the centralizer as module over the polynomial ring in one variable; it also yields the equation of the spectral curve of the centralizer and an explicit presentation of the rank-2 vector bundle consisting of common eigenfunctions, via (sub)resultants. The theory is based on the algebro-geometric interpretation of the objects of study in terms of the spectral curve. Klein's quartic curve is not capable of such an interpretation, and a generalization of the theory is devised, that produces commutative rings of matrix ODOs (cf. [2]). We then pursue the study of commutative rings of matrix ODOs with polynomial coefficients. This is work in collaboration with Sonia L. Rueda and Maria-Angeles Zurro.

Keywords
Weyl Algebra, Rank of an Algebra of Differential Operators, Klein Quartic Curve

References
[1] J. DIXMIER, Sur les algèbres de Weyl. *Bull. Soc. Math. France* **96**, 209–242 (1968).
[2] I.M. KRICHEVER, Algebraic curves and commuting matrix differential operators. *Funkcional. Anal. i Priložen*. **10**(2), 75–76 (1976).
[3] E. PREVIATO; S.L. RUEDA; M.-A. ZURRO, Commuting ordinary differential operators and the Dixmier test. *SIGMA Symmetry Integrability Geom. Methods Appl.* **15**(Paper No. 101), 23 pp. (2019).