Applications of Computer Algebra – ACA 2021
Virtual. Online | July 23-27, 2021
Session on *"Algebraic and Algorithmic Aspects of Differential and Integral Operators (AADIOS)"*

Factoring Third Order Ordinary Differential Operators over Spectral Curves

Sonia L. Rueda[1]***, Maria-Angeles Zurro***[2] [sonialuisa.rueda@upm.es]

[1] Dpto. de Matemática Aplicada. Universidad Politécnica de Madrid. Spain
[2] Dpto. de Matemáticas. Universidad Autónoma de Madrid. Spain

We consider the factorization problem of a third order ordinary differential operator $L - \lambda$, for a spectral parameter λ and an irreducible operator L, whose coefficients belong to a differential field K. It is assumed that L is algebro-geometric over K, guarantying a nontrivial centralizer, which can be seen as the ring of an affine curve, the famous *spectral curve* Γ.

Based on the nature of Γ, we give a symbolic algorithm to factor $L - \lambda$ over the spectral curve using differential subresultants. In this context, the first explicit example of a non-planar spectral curve arises, as well as the factorization it provides for $L - \lambda$. As far as we know, it is the first factorization algorithm for third order irreducible operators over the field extension $K(\Gamma)$ of K. The coefficient field K is extended to the field of rational functions on Γ to obtain a right factor with coefficients in this field.

Factorizations over planar spectral curves have been presented in other articles, for instance for second order operators [1], or fourth order operators with rank 2, [3]. The present work is the natural continuation in a program dedicated to the factorization of rank 1 algebro-geometric differential operators, that was already successful in the order 2 case, [1]. Our ultimate goal is an effective approach to the direct spectral problem and the development of the appropriate *spectral Picard-Vessiot fields* containing all the solutions of the operator $L - \lambda$. Spectral Picard-Vessiot fields were studied for Schrödinger operators in [2].

Keywords
Factorization, ordinary differential operators, differential subresultant, spectral curve.

References

[1] J.J. MORALES-RUIZ. S.L. RUEDA, AND M.A. ZURRO. *Factorization of KdV Schrödinger operators using differential subresultants*. Adv. Appl. Math., 120:102065, 2020.

[2] J.J. MORALES-RUIZ. S.L. RUEDA, AND M.A. ZURRO. *Spectral Picard-Vessiot fields for algebro-geometric Schrödinger operators* . To appear in Ann. Inst. Fourier. Arxiv https://arxiv.org/abs/1708.00431, 2021.

[3] E. PREVIATO, S.L. RUEDA, AND M.A. ZURRO. *Commuting Ordinary Differential Operators and the Dixmier Test*. SIGMA Symmetry Integrability Geom. Methods Appl., 15(101):23 pp., 2019.