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Integral bases of P-recursive operators

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Integral bases are a central concept when studying algebraic field extensions, with applications including the computation of short generators or symbolic integration [2]. In recent work [1], similar bases were defined over differential Ore algebras. They can be computed with a generalization of an algorithm from the algebraic case [3], and they have the same application to the computation of integrals.

In this work, we explain how to extend the definition and the construction to other Ore algebras, such as those defined with a "shift" operator. More generally, we extend the definition to valued vector spaces satisfying some additional axioms. This setting contains the known cases of algebraic extensions and differential Ore algebras, and we show that shift Ore algebras can be equipped with a suitable valuation. As a result, we obtain a general framework for defining and computing integral bases.

Keywords
Ore algebra, Shift operators, integral bases

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