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The Newton-Puiseux Algorithm and Effective Algebraic Series

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Given a polynomial $p(x, y)$ in two variables x and y over an algebraically closed field $\mathbb{K}$ of characteristic zero, the classical Newton-Puiseux algorithm [2] determines the first terms of a series ϕ in x over $\mathbb{K}$ that solves $p(x, \phi) = 0$. Finding a series solution of a polynomial equation is one and the most apparent aspect of the algorithm. However, it also permits to encode algebraic series by a finite amount of data and to effectively compute with them on the level of these encodings. While this is well-known for univariate algebraic series, this is not the case for algebraic series that are multivariate. We explain how to do effective arithmetic with multivariate algebraic series and complement the discussion of the Newton-Puiseux algorithm for (multivariate, not necessarily bivariate) polynomials over a field of characteristic zero in [3]. We also show that the convex hull of the support of an algebraic series is a polyhedral set and explain how the Newton-Puiseux algorithm and an effective equality test for algebraic series can be used to compute its vertices and bounded faces.

Keywords
Newton-Puiseux algorithm, algebraic series, effective arithmetic, supports of series

References
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