Applications of Computer Algebra – ACA 2023
Warsaw, Poland, | July 17-21, 2023
Session on *"Algebraic and Algorithmic Aspects of Differential and Integral Operators (AADIOS)"*

Computation of Koszul homology and application to involutivity of partial differential systems

Cyrille Chenavier[1]***, Thomas Cluzeau***[1]***, Alban Quadrat***[2] [cyrille.chenavier@unilim.fr]

[1] XLIM, Université de Limoges, Limoges, France
[2] Institut de mathématiques de Jussieu-Paris rive gauche, Université Sorbonne, Paris, France

This talk will be a presentation of the paper [1]. The formal integrability of systems of partial differential equations plays a fundamental role in different analysis and synthesis problems for both linear and nonlinear differential control systems. Following Spencer's theory, to test the formal integrability of a system of partial differential equations, we must study when the symbol of the system, namely, the top-order part of the linearization of the system, is 2-acyclic or involutive, i.e., when certain Spencer cohomology groups vanish. Using the well-known fact that Spencer cohomology is dual to Koszul homology and symbolic computation methods, we show how to effectively compute the homology modules defined by the so-called Koszul complex of a finitely presented module over a commutative polynomial ring. These results are implemented using the OREMORPHISMS package. We then use these results to effectively characterize 2-acyclicity and involutivity of the symbol of a system of partial differential equations. Finally, we show explicit computations on different examples.

Keywords
formal integrability, involutivity, Koszul complex

References
[1] C. CHENAVIER; T. CLUZEAU; A. QUADRAT, *Computation of Koszul homology and application to involutivity of partial differential systems*. IFAC Symposium on System Structure and Control (SSSC 2022).