

# Crossed homomorphisms and Cartier-Kostant-Milnor-Moore theorem for difference Hopf algebras

*Li Guo*<sup>1</sup>, *Yunnan Li*<sup>2</sup>, *Yunhe Sheng*<sup>3</sup>, *Rong Tang*<sup>3</sup>

[[liguo@rutgers.edu](mailto:liguo@rutgers.edu)]

<sup>1</sup> Department of Mathematics and Computer Science, Rutgers University, Newark, NJ 07102, USA

<sup>2</sup> School of Mathematics and Information Science, Guangzhou University, Guangzhou 510006, China

<sup>3</sup> Department of Mathematics, Jilin University, Changchun 130012, Jilin, China

The celebrated Milnor-Moore theorem and the more general Cartier-Kostant-Milnor-Moore theorem establish close relationships of a connected and a pointed cocommutative Hopf algebra with its Lie algebra of primitive elements and its group of group-like elements. Crossed homomorphisms for Lie algebras, groups and Hopf algebras have been studied extensively, first from a cohomological perspective and then more broadly, with an important case given by difference operators. In this talk we show that the relationship among the different algebraic structures captured in the Milnor-Moore theorem can be strengthened to include crossed homomorphisms and difference operators. We give a graph characterization of Hopf algebra crossed homomorphisms which are also compatible with the Milnor-Moore relation. We further investigate derived actions from crossed homomorphisms on groups, Lie algebras and Hopf algebras, and establish their relationship. Finally we obtain a Cartier-Kostant-Milnor-Moore type structure theorem for pointed cocommutative difference Hopf algebras. Examples and classifications of difference operators are also provided for several Hopf algebras.

## Keywords

crossed homomorphism, difference operator, Hopf algebra, Lie algebra, Milnor-Moore theorem, Cartier-Kostant-Milnor-Moore theorem