

# A Semi-decision Procedure for Proving Operator Statements

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Linear operators play a fundamental role in various mathematical contexts, appearing as ring elements (e.g., in  $C^*$ -Algebras), as (rectangular) matrices, or as vector space and module homomorphisms. In this talk, we present a recently developed algebraic framework [1] for proving first-order statements about linear operators by computations with noncommutative polynomials. Furthermore, we present our new SAGEMATH package `operator_gb` [2], which offers functionality for automatising such computations. We aim to provide a practical understanding of our approach and the software through examples, including recent work [3], while also explaining the completeness of the method in the sense that it allows to find algebraic proofs for every true first-order operator statement. Our main result is a semi-decision procedure that allows to automatically prove operator statements phrased within first-order logic based on a single computation with noncommutative polynomials.

## Keywords

linear operators, first-order statements, noncommutative polynomials

## References

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