

Difference-Differential Polynomials in SageMath

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In this talk we are going to present `dalgebra`, a new module developed for SageMath [7] focused on the description of the structures and elements necessary to work in the fields of Differential or Difference Algebra.

A differential ring [6] is a pair (R, ∂) where R is a ring and ∂ a derivation over R . Similarly, a difference ring is a pair (R, σ) where now σ is a ring homomorphism. From these rings we can build difference/differential extensions, we can state respectively summation or integration problems, we can try to solve difference or differential systems, etc. In particular, there is a particular extension of particular interest for us: the ring of difference/differential polynomials.

There are many problems in both the differential and the difference world, in some cases even equivalent problems: as an example we can consider the Symbolic Integration problem [3] and Symbolic Summation problem [8]. We found that a generic software to work with these objects is not easy to find. There are implementations for linear operators (both with differences and derivation) in SageMath (`ore_algebra` [4]), in Maple (`OreTools` [1]), and in Mathematica (`HolonomicFunctions` [5]). We can also find an implementation of differential polynomials (not difference) in the Maple package `DifferentialAlgebra` [2].

This is the reason we decided to implement a new package in SageMath [7], an open source Computer Algebra software based on Python. This new package `dalgebra` provides a simple framework to define difference and differential ring and even a combination of the two. It also implements the difference and differential polynomials. This is a great starting point to implement further algorithms in the world of difference and differential algebra.

The package `dalgebra` is publicly available on Github* and is under active development, adding more features and improving the user interface. In this talk we will present the main features of `dalgebra`, including but not being limited to how we handle differential rings, how to set up a system of such equations and how to manipulate differential polynomials.

Keywords

difference algebra, differential algebra, SageMath, linear operators

*<https://github.com/Antonio-JP/dalgebra>

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