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Towards an effective integro-differential elimination theory

***Thomas Cluzeau*[1], *Camille Pinto*[2], *Alban Quadrat*[2]** [camille.pinto@inria.fr]

[1] Université de Limoges, XLIM, Limoges, France
[2] Inria Paris, IMJ–PRG, Paris, France

Algebraic analysis is a mathematical theory which studies linear systems of ordinary or partial differential equations using rings of partial differential operators, module theory, homological algebra etc. Within this approach, a linear functional system yields a finitely presented left module over a non commutative polynomial ring of functional operators. Structural properties and equivalences of linear systems can be intrinsically reformulated within module theory and homological algebra. A classic environment in algebra is a noetherian ring with finitely generated modules. An example is the Weyl algebra $\mathbb{A}_1$: the ring of differential operators with polynomial coefficients in one variable (coefficient living in the commutative field $\mathbb{k}$). But some algebras are not noetherian, such as the algebra of entire functions, or the algebra of polynomial in an infinity of variable.

Our team research is currently interested in studying rings of integro-differential operators. The use of this rings allows one to algebraize elementary calculus by combining the differential operator, the indefinite integral and the evaluation at the initial time t_0. $\mathbb{I}_1$ usually denotes the ring of integro-differential operators in one variable with polynomial coefficients. It can be define by: « the smallest $\mathbb{k}$-algebra containing t (the operator define by the product with the elementary polynomial t), ∂ (the differential operator), and I (the integral operator) ».
Bavula worked on this subject recently (see [1]) and proved that $\mathbb{I}_1$ was not noetherian. Nonetheless, Bavula also proved that $\mathbb{I}_1$ was a coherent algebra. That means that all finitely generated sub-module of $\mathbb{I}_1$ is finitely presented. Adding a condition on sub-modules offsets the non-notherianity of $\mathbb{I}_1$ and allows us to do calculus. Yet, Bavula gave a theorical argument to say that $\mathbb{I}_1$ is coherent. We would like to make this proof effective and be able to actually make calculus in $\mathbb{I}_1$ with a computer machine.

Keywords
Integro-differential operator, Annihilator, Coherence

References
[1] V. V. BAVULA, Title. *Journal of Pure and Applied Algebra* **217**(3), 495–529 (2013).