

# An abelian ambient category for behaviors in algebraic systems theory

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In the algebraic analysis approach to systems theory, systems are modeled as solution sets of finitely many linear equations over a ring, usually a ring of differential operators. These solutions are taken within some fixed module, usually a module whose elements may be interpreted as trajectories. Willems coined the term *behavior* for such solution sets [1]. Whenever our ring is noetherian and our fixed module is an injective cogenerator, a good notion of an abelian ambient category for behaviors is known in algebraic systems theory: namely the opposite category of finitely presented modules over our ring. In that good case, intrinsic features of behaviors, like being controllable or autonomous, translate into intrinsic features of finitely presented modules, like being torsion-free or torsion.

We propose a setup for algebraic systems theory that also works for an arbitrary fixed module over an arbitrary ring [2]. This setup is based on functor categories instead of module categories. This functorial setup overcomes some deficiencies that arise within the module theoretic approach, like behaviors not being isomorphic in situations where they clearly should be isomorphic. We provide an example study case with delay-differential systems.

## **Keywords**

Algebraic systems theory, delay-differential systems, module-behavior duality, abelian categories, finitely presented functors

## **References**

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