Faster multivariate integration in D-modules

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Abstract

Not all integrals can be expressed in closed form using elementary functions, as shown by Liouville's theorem. In contrast, the integral of a holonomic/D-finite function is always holonomic/D-finite, that is the integral of a function satisfying sufficiently many linear differential equations (LDEs) with polynomial coefficients also satisfies such a system of LDEs. This makes the holonomic and D-finite frameworks particularly relevant for symbolic integration.

I will address two central algorithmic problems in this field: the problem of integration with parameters, where one seeks a differential equation satisfied by a parametric integral, and the reduction problem, where the goal is to find linear relations between integrals. Two distinct approaches exist, the D-finite one and the holonomic one. The D-finite approach has been the most studied one and offers efficient algorithms, but it lacks the full expressivity of the holonomic setting, which can handle a broader class of integrals and particularly those over semi-algebraic sets. However, the current algorithms developed for the holonomic setting have a prohibitive computational cost. I will present a new reduction algorithm working in a mixed approach, aiming to balance the efficiency of D-finiteness with the expressivity of holonomy. This reduction is inspired by the Griffiths–Dwork method for rational functions [1, 2] and yields similarly an algorithm for the problem of parametric integration.

As an application, I will present the computation of a differential equation for the generating function of 8-regular graphs, which was out of reach so far.

This work was conducted jointly with my PhD advisors, Frédéric Chyzak and Pierre Lairez, and is based on our paper [3].

References

[1] A. Bostan, P. Lairez, B. Salvy. Creative telescoping for rational functions using the Griffiths-Dwork method. Proceedings of the 38th International Symposium on Symbolic and Algebraic Computation, 93-100, 2013.

[2] P. Lairez. Computing periods of rational integrals. Mathematics of computation 85, pp. 1719-1752, 2016.

[3] H. Brochet, F. Chyzak, P. Lairez. Faster multivariate integration in D-modules. Preprint, 2025.