Hypergeometric solutions of elliptic difference equations

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Abstract

In this presentation, we will present an algorithm to compute hypergeometric solutions of a linear difference equation on an elliptic curve.

1 Introduction

Consider an elliptic curve \mathcal{C} with coefficients in $\overline{\mathbb{Q}}$ and $\delta \in \mathcal{C}(\overline{\mathbb{Q}})$ a non torsion point. We consider an elliptic difference equation $\sum_{i=0}^{l} a_i(p) f(p \oplus i.\delta) = 0$ with \oplus the elliptic addition law and a_i polynomials on \mathcal{C} . We present an algorithm to compute rational solutions, then an intermediary class we call pseudo-rational solutions, and finally hypergeometric solutions, which are functions f such that $f(p \oplus \delta)/f(p)$ is rational over \mathcal{C} .

References

[1] Thierry Combot. Hyperexponential solutions of elliptic difference equations, 29 Apr 2022. https://arxiv.org/abs/2205.00041.