An Effective Version of the Grothendieck p-curvature Conjecture for Order One Differential Equations

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To a linear differential equation with polynomial coefficients over the rational numbers one can attach, for all prime numbers p, a linear map called the p-*curvature*. The Grothendieck p-curvature conjecture asserts that the algebraicity of a full basis of solutions of such a differential equation is equivalent to the vanishing of the p-curvatures for almost all prime numbers p. In 1974 Honda provided a proof of this conjecture for order one equations [2], by reducing the problem to a theorem of Kronecker [3], which provides a local-global criterion for the splitting of polynomials over the rational numbers. In 1985 Chudnovsky and Chudnovsky gave a new proof of Kronecker's result [1], and with it of Honda's result, using Padé approximation.

In this talk I will explain how to use the proof of the Chudnovsky brothers to make Honda's result effective. More precisely, given a linear differential equation of order one with polynomial coefficients over the rational numbers we deduce an upper bound on the number of p-curvatures to be computed in order to decide the algebraicity of all solutions of the equation.

This talk is based on ongoing joint work with Lucas Pannier.

References

[1] David V. Chudnovsky and Gregory V. Chudnovsky. Applications of Padé approximations to the Grothendieck conjecture on linear differential equations. *Number theory* (New York, 1983–84). Vol. 1135. Lecture Notes in Math. Springer, Berlin, 1985, pp. 52–100.

[2] Taira Honda. Algebraic differential equations. *Symposia Mathematic.*, Vol. XXIV (Sympos., INDAM, Rome, 1979). London-New York: Academic Press, 1981, pp. 169–204.

[3] Leopold Kronecker. Über die Irreducibilität von Gleichungen *Monatsberichte der Königlichen Preussische Akademie des Wissenschaften zu Berlin* (1880), pp. 155–163.