An algorithmic problem for Nijenhuis Lie algebras

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Abstract

In this work, we address an algorithic problem for Nijenhuis Lie algebras. We introduce the concept of HNN-extension for Nijenhuis Lie algebras and employ the Gröbner-Shirshov basis theory for free Nijenhuis Lie algebras to provide an embedding theorem.

1 Introduction

The role of the Nijenhuis operator on a Lie algebra has been used in the study of integrability of nonlinear evolution equations in [1]. In this work, we spread the concept of HNN-extension which is an important construction in combinatorial group theory to free Nijenhuis Lie algebras. HNN-extension has been spread to various algebraic structures such as Lie (super)algebras, Leibniz algebras, semigroups, and rings. The following presentation exists for HNN-extension of Lie algebra \mathcal{L} :

$$\mathcal{H} = \langle \mathcal{L}, t \mid [t, a] = d(a), \text{ for all } a \in \mathcal{A} \rangle, \tag{1}$$

where d is a derivation map defined on a subalgebra \mathcal{A} and t is a new generating letter. We develop this construction to the case of free Nijenhuis Lie algebras. To this end, we recall the theory of Gröbner-Shirshov basis for Lie Ω -algebras introduced in [3] and provide a presentation for HNN-extension of free Nijehuis Lie algebras. As for an application of HNN-extension, we provide an embedding theorem. It is noting that HNN-extension provides alternative proofs for known embedding theorems, and used in undecidability of Markov properties (see [2] and references herein).

References

- [1] I. Dorfman. Dirac Structures and Integrability of Nonlinear Evolution Equations. (Wiley, Chichester, 1993).
- [2] A. Najafizadeh, C. Zargeh On the undecidability of Markov properties for Lie superalgebras. J. Algebra Comb. Discrete Appl. 12(1):43-52. 2025.
- [3] J. Qiu, Y. Chen. Groebner-Shirshov bases for Lie Ω-algebras and free Rota-Baxter Lie algebras. J. Alg. Appl., 16(2):1750190, 2017.