

# Foreword

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The arsenal of symbolic methods for differential equations is continuously expanded and refined, and along with them a plethora of diverse data structures, conceptual frameworks, and theoretical results. In this Special Issue, we have adopted a unifying perspective from the algebraic operator vantage point. We believe that this viewpoint offers a fruitful basis for present and future research, aiming at a deeper understanding of differential and integral operators.

With this goal in mind, the first two of us organized special sessions at the annual international conference on *Applications of Computer Algebra* (ACA). Three Special Sessions, under the title *Algebraic and Algorithmic Aspects of Differential and Integral Operators Session* (AADIOS) of ACA were held: at the Castle of Hagenberg, Austria (July 2008), in Montreal, Canada (June 2009), and in Vlora, Albania (June 2010). In fact, most papers in this Special Issue have been presented—at least in part—in these sessions.

The scope covered by these presentations/papers conforms in general to the list of topics envisioned in the Call for Papers for the Special Sessions and this Special Issue:

- Symbolic Computation for Operator Algebras
- Factorization of Differential/Integral Operators
- Linear Boundary Problems and Greens Operators
- Initial Value Problems for Differential Equations
- Symbolic Integration and Differential Galois Theory
- Symbolic Operator Calculi
- Algorithmic D-Module Theory
- Rota-Baxter Algebra
- Differential Algebra
- Discrete Analogs of the above
- Software Aspects of the above

A quick look through the Table of Contents will reveal that all of these topics have actually found their expression here. The 14 articles collected in this double Special Issue (listed in chronological order of submission date) exhibit a great variety of approaches and methods. They range from rather algebraic treatments all the way to technical system descriptions. Specific packages are mentioned in almost all of the papers. All papers have been subjected to a thorough refereeing process (2-4 referees per paper), with an explicit second round of reviews after revisions according to suggestions and comments of the referees.

In differential algebra, A. Levin studies how differential birational invariants for differential field extension carried by its differential dimension polynomial are related to two new concepts called transcendental type and dimension. Under rather general assumptions, these concepts provide another interpretation, in terms of generic solutions, of what A. Einstein described as the strength of a differential system.

The paper by S. Morikawa, K. Saito, T. Takeuchi and H. Umemura offers an unorthodox view of Burgers' equation  $u_t = 2uu_x + u_{xx}$ . They analyze an ultra-discretized model over finite fields and its differential Galois group. They also illustrate their results with examples from computer science (cellular automata), religious art (Sistine Chapel, Mandala), and nature (clamshells).

A. Kasman and E. Previato study commutative rings of partial differential operators, exploring factorization, differential resultants, quantization, conjugation and the transference method of Burch-nall and Chaundy. In addition to offering downloadable procedures suitable for testing conjectures, they present a bank of geometrically interesting examples, some open problems, and challenges to the research community.

F. Boulier and F. Lemaire give a new algorithm for computing the normal form of a differential rational function defined on a differential variety whose defining differential ideal is given by a regular differential chain. The algorithm is applied to compute formal power series solutions up to a given order of approximation.

The paper by A. F. Cheviakov presents a computer algebra package for the computation of local symmetries of differential equations, particularly point symmetries of linear partial differential systems. Computer algebra is used for systems with parameters. Applications include splitting, simplifying, and solving the computed local symmetries.

E. Shemyakova considers a method that refines a pair of two-factor factorizations of a linear partial differential operator into a three-factor one, and generalizes it from low order cases in two derivations to the case for arbitrary order and number of derivations. Many examples are given and the paper may also serve as a tutorial for related factorization methods.

W. Plesken and D. Robertz study the question of expressing an analytic function as a linear combination of given analytic functions whose coefficients are restricted to domains that are specified as images of some given functions. The existence of a generic representation can be decided (and computed if it exists) algorithmically by computing a certain left ideal in a ring of differential operators.

Mikusinski's classical operational calculus is generalized to nonlocal Cauchy boundary problems in the paper by I. Dimovski and M. Spiridonova, which leads to an extension of the Duhamel principle. Applications include solutions of periodic, antiperiodic and mean-periodic LODEs.

Various improvements of the creative telescoping method for definite integration and summation in differential-difference operator algebras are described in the paper by C. Koutschan. The implementation exhibits good timings and enables the treatment of problems that were deemed practically impossible for other packages.

The paper by M. A. Barkatou, E. Pflügel and G. Broughton deals with the local analysis of systems of ordinary linear differential, difference, and  $q$ -difference equations in a unified theoretical framework. A generic algorithm for computing a basis of regular solutions is presented and demonstrated by the aid of a dedicated package.

M. S. Boudellioua and A. Quadrat present a new constructive approach to Serre's reduction, which is a method for finding equivalent presentations of linear functional systems (which subsume partial differential, difference, and time-delay systems) using fewer equations and unknowns. The paper also contains several illuminating concrete examples computed with packages by the authors.

In their two papers, L. Guo and W. Y. Sit study the enumeration and the generating functions of (differential) Rota-Baxter words, which are crucial for the combinatorial study of free (differential) Rota-Baxter algebras and a foundation for implementing algorithms on expressions like nested integrals. Differential Rota-Baxter algebras, which encompass differential and integral operators in a unified structure, offer a promising platform for the algebraic operator vantage point advocated in this Special Issue.

In the paper by D. Andres, M. Brickenstein, V. Levandovskyy, J. Martín-Morales and H. Schönemann, algorithms from D-module theory are reviewed and illustrated on concrete examples using

the authors' implementation. In particular, they present a new approach for computing Bernstein operators and Bernstein-Sato polynomials and related objects.

As we can now all enjoy the fruits of our labor, it is our pleasure to acknowledge freely that we have received help and encouragement from many sources—we are indeed grateful to them all. First of all, we would like to thank Dongming Wang for his spontaneous agreement to publish this Special Issue and for his ongoing guidance and support. Of course we extend our thanks to all authors for their trust and effort (particularly throughout the reviewing rounds), to the referees for their diligence and patience, and to the AADIOS participants for their enthusiasm and feedback. At this juncture, we thank also the ACA organizers and ACA Working Group for always providing support for our sessions. The informal character of these sessions, with lively discussions often spilling into coffee breaks and continuing at social events, makes the ACA venue ideal for presenting also “work in progress”. This Special Issue in *Mathematics in Computer Science* serves as a lucid testament to the active research that had been presented at these AADIOS sessions.

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